CERTAIN ASPECTS OF THE PROBLEM OF REDUCING THE FORCE OF GRAVITY

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(NASA-TT-F-15839) CERTAIN ASPECTS OF THE PROBLEM OF REDUCING THE FORCE OF GRAVITY (Techtran Corp.) 11 p HC $4.00 CSCL 08E

N74-30704

Unclas
G3/13 45735

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JULY 1974

STANDARD TITLE PAGE

1. Report No. NASA TT F-15,839	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle CERTAIN ASPECTS OF THE PROBLEM OF REDUCING THE FORCE OF GRAVITY		5. Report Date JULY 1974
		6. Performing Organization Code
7. Author(s) S. V. Yevseyev		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Techtran Corporation P.O. Box 729 Glen Burnie, Maryland 21061		11. Contract or Grant No. NASw-2485
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: "O Nekotorykh Voprosakh Problemy Reduktsiy Sily Tyazhesti," Geofizicheskiy Sbornik, Volume 55, 1973, pp. 3-8

16. Abstract

The Faye-Putnam reduction formula was deduced by the author on the basis of the established relation between anomalies and regional topography and again by Taylor's series expansion.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified-Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 10	22. Price

CERTAIN ASPECTS OF THE PROBLEM OF REDUCING THE FORCE OF GRAVITY

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The correlation between gravitational anomalies and so-called "regional topography" — i.e., relief of a location around an observation station — has been the subject of study of many researchers. /3*

Shortly after the research by the well known scientists Erie (1885) and Pratt (1856) on the principle of isostatic compensation the French astronomer and geodesicist Faye (1883) concluded that the observed values of the force of gravity, with correction for normal values for the force of gravity and corrections made for altitude (the so-called correction "in free air"), should be also made for the attraction of an infinitely dense parallel stratum whose thickness is equal to the difference between the altitude station above sea level h and the mean altitude of the locale surrounding the station H

$$\Delta g_F = \Delta g - 2\pi f \mu (h - H_r); \tag{1}$$

Here Δg — anomaly "in free air", f — gravitation constant, μ — density of relief mass, H_r — mean altitude of locale surrounding a station having a certain radius r.

In fact, the American geodesicist Patten (1894) named this anomaly Δg_F Faye's anomaly and not the anomaly "in free air" as is often the case. From the viewpoint of the isostatic theory, the expression $2\pi f\mu(h-H_r)$ can be viewed as a formula for isostatic reduction in a simplified form [4].

It will be shown further how one can approach formula (1) by another path. Now we shall examine what consequences devolve from it. In the ideal case, in the presence of a perfect isostasis, Δg_F will equal zero, and consequently,

$$\Delta g = 2\pi f \mu (h - H_r). \tag{2}$$

If the region of application (2), (the so-called zonal region) is limited in area and the altitudes of zonal relief vary weakly, then obviously, mean

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*Numbers in the margin indicate pagination in the foreign text.

altitude H_r from point to point will change quite slowly and only slightly such that magnitude $-2\pi f\mu H_r$ can be accepted as the constant a with some error:

$$\Delta g = a + bh. \qquad (3)$$

Most investigators sought the correlational relationship between anomalies "in free air" and altitudes of relief [8, 15, 17] in such a form. For this purpose, from the system of equations having formed (3), and assuming that Δg and h are known, one finds a and b by the method of least squares. This method is attractive by reason of its simplicity, however, it has a limited sphere of application since parameter a, accepted as a constant value, in reality can noticeably change in certain places on the strength of various reasons. Therefore, determination of parameters a and b from any aggregation of points arbitrarily located relative /4 to the surface and altitudes of any arbitrarily selected region is a principal error. Studies demonstrate that determinable and rigid conditions must be observed during this process — uniform distribution of points relative to altitude and area [4]. An incorrect method lowers the accuracy of determining the parameters and occasionally leads to obtaining values of the latter devoid of meaning, inasmuch as error can reach 100% of the determined value, as theoretical examples demonstrate. Therefore, the value of coefficient b in the selected regions (usually trapezoids, limited by parallels and meridians) has been obtained by different investigators in differing values. For example, Jeffreys' values [17] for continental regions fluctuate within limits from $+0.182\ \frac{\text{mgl}}{\text{m}}$ to $-0.008\ \frac{\text{mgl}}{\text{m}}$ [17], those of Zhongolovich from $+0.121\ \frac{\text{mgl}}{\text{m}}$ to $-0.011\ \frac{\text{mgl}}{\text{m}}$ (in the northern hemisphere) and from $+0.399\ \frac{\text{mgl}}{\text{m}}$ to $+0.007\ \frac{\text{mgl}}{\text{m}}$ (in the southern hemisphere) [8]. The distribution of values of b itself on the Earth's surface has a random and nonpolar character for these investigators. For the very same places on the Earth, the indicated authors had values of b which were dissimilar. The reasons for this, in all probability, could be an inadequate number of gravitation points, and their unfavorable location — predominantly in low places, etc. It has not been explained how coefficient b can have negative significance in purely continental regions[2].

[2] For example, $b = -0.011\ \frac{\text{mgl}}{\text{m}}$ in Texas, USA; $b = -0.007\ \frac{\text{mgl}}{\text{m}}$ in Kazakhstan, etc. [8].

The results of our investigations, which were begun in 1939 and formulated in 1949 and 1957 [2, 4] show that when observing certain conditions, coefficient b must have a value which is constant and proximate for all continental regions, i.e., near value $2\pi f\mu$ — the Bouguer coefficient of reduction which is equal to 0.0419μ for land and $0.0419\ (\mu'-\mu)$ for the sea, where μ' is the density of sea-water. Thus, our investigations conducted at a number of high mountainous regions of the Earth — the Pamir, Altay, the Caucasus, the Crimea, and Hawaii — demonstrate that the value of coefficient b for land fluctuates within limits of $0.009\ \frac{\text{mgl}}{\text{m}}$ to $0.011\ \frac{\text{mgl}}{\text{m}}$, which corresponds to $2\pi f\mu$ with a value $\mu = 2.6\text{-}2.4$ [2, 4].

These conclusions were checked in 1967 by B. L. Skuin on the territory of a mountainous region of the Ukraine whose area comprised nearly 8,000 square kilometers and where nearly 2,000 points were located and equally distributed over a distance of 2.0-2.5 km from each other. The high accuracy of the original material, the gravimetric survey, and the uniform and extremely dense locations of points with respect to area and altitude provided high reliability of calculations in this "experimental polygon". The results of the investigations of B. L. Skuin fully confirmed our conclusions [11]. The same results were reached abroad in 1957 by Wotil [16] and in the Soviet Union in 1962 by Taranov [13] and others.

Directly related to this is the question of "expression" of anomalies of the force of gravity when it becomes necessary to find mean anomalies in free air with respect to individual regions (sectors, trapezoids) of the Earth's surface.

Proceeding from formula (3), we find that mean anomaly Δg equals

$$\overline{\Delta g} = a + b\overline{H}, \tag{4}$$

where $\overline{H}$ is the mean altitude of the given region. Many investigators employed this formula but the principal difference of our formula [4] from the similar formula of Jeffreys consists in the fact that in our formula coefficient b was constant everywhere and equalled $2\pi f\mu$, while at the same time it varied among other investigators. For example, Zhongolovich, in finding the anomalous mid-point by the aid of equality (4) used values b found by him from any points, occasionally an insignificant number, and found at random within the /5 limits of any particular trapezoid into which he divided the Earth's surface. B. L. Skuin, in order to study this question, isolated 48 regions in the "test polygon" described above and within the limits of this polygon took half the points located in high places and deliberately excluded them from treatment, thereby artificially creating a situation existing in many places of the Earth's surface [11]. The mean anomaly was found twice: with $b = 2\pi f\mu$ and at values of b calculated from the remaining point in each region. The results were compared with the mean anomaly found from the total of all points. It proved that in the first method, i.e., with $b = 2\pi f\mu$, mean quadratic error was nearly 3 times less than it was in the second method. Therefore the result of calculation obtained by Jeffreys and Zhongolovich — dimensions of a terrestrial ellipsoid and parameters of the formula of the normal force of gravity — seem doubtful.

If one now moves from anomalies "in free air" to the Bouguer anomalies, then on the basis of formula [2]

$$\Delta g_B = -2\pi f\mu H_r. \tag{5}$$

This relationship was first stated by us in 1949 based on the material of anomalies in certain mountainous regions [2], and was confirmed in 1952 and 1957 [4, 3]. In 1967, a graph of relationships of mean Bouguer anomalies and mean region altitudes located in 6 most characteristic continental and maritime regions of the Earth was compiled; during this process the boundaries of regions were selected by us insofar as possible in accordance with the tectonic and geomorphological conditions. In the constructed graph, the points corresponding to these regions coincided well enough with line $2\pi f\mu H$ [5].

In 1961, Grushinskiy, in investigating this relationship on the scale of the entire Earth, established that coefficient b is 0.096 $\frac{\text{mgl}}{\text{m}}$ for land and -0.070 $\frac{\text{mgl}}{\text{m}}$ for the sea [1] as one would except for the case of averaging. Bouguer anomalies and altitudes by five-degree trapezoids.

For the general case one can assume that

$$\Delta g_B = c - 2\pi f\mu H_r, \tag{6}$$

where c is a certain value near the constant within limits of a zonal region and depends upon different factors — error of the normal formula used in calculations, inaccuracy of coefficients, regional disorders in the isostatic compensation, etc.

The results of the investigations of B. L. Skuin carried out in 1971 on the polygon described above confirmed our conclusions. From the solution of conditional equations of type (6) in 274 regions, coefficient b proved equal to 0.01 $\frac{\text{mgl}}{\text{m}}$, with r = 50 km. During this process, mean quadratic error was almost 2.5 times less than where r = 0, i.e., if one takes the individual altitudes of points [12], since certain investigators assume that Δg_B depends in fact on the very altitudes of observation stations, although a wide dispersion of points in the graphs occurs during this process [5].

The Investigations of the geophysicist Mabey demonstrated that in 2 regions /6 of North America — the states Nevada and Idaho — the Bouguer anomaly correlates well with mean altitudes with a radius magnitude of region equal to r = 64 km [18]. The coefficient with H_r proved to be 0.10 $\frac{\text{mgl}}{\text{m}}$, in accordance with the material presented above. Formula (2) for anomalies in free air can also be presented in a more generalized form

$$\Delta g = 2\pi f \mu (h - H_r) + c. \tag{7}$$

The coefficient with $(h-H_r)$ calculated by us in 1957 for regions of the Himalayas and the Pamir also proved to be 0.11-0.10 $\frac{\text{mgl}}{\text{m}}$ with radius r = 1.5° and 0.5°, respectively [4]. For B. L. Skuin, for the above described mountainous region, this also was 0.11 $\frac{\text{mgl}}{\text{m}}$ with radius r = 50 km [11].

Formulas (1) and (2) were investigated by V. A. Litinskiy, A. V. Ladynin and other geophysicists who successfully employed them for geophysical interpretation [9, 10]. A. V. Ladynin did much valuable work, particularly with respect to the problem of finding the optimal value of the "radius of regionality". But this is not to negate the work of the other authors, who found it vital to introduce into the anomaly the so-called "correction for orography", whose value is not insignificant. Therefore, in the mountain regions of the Carpathians it is equal, in approximately half of the cases, to 5 mgl, and in certain points can reach 25 mgl. What is important is that calculating this correction requires almost as much work as calculating the isostatic reduction of plane zones. In this or any other case, it is necessary to find the altitude in relative trapezoids of plane zones of Hayford, using large scale maps, up to 167 km from the point. Moreover, there is an undiscovered reduction of far and so-called spherical zones which can reach truly large values [7]. The proponents of the anomaly expressed by formula (1), assume [9] that one of its principal advantages is the fact that neither its value nor the method of calculation depend upon the system of making compensation. Therefore the value of coefficients in the topographic (with h) and compensative (with H) parts are accepted as equal and equal to $2\pi f\mu H_r$. If this is more or less justified for the coefficient of topographical reduction, matters stand entirely differently for reduction for compensation. It is primarily necessary to bear in mind that derivative $2\pi f\mu H_r$ expresses the extent of a stratum of infinite expanse reaching thickness H_r. But under conditions of a regional character of compensation, with limited dimensions of the region of compensation, the value of the coefficient will depend upon the "radius of regionality" r and generally speaking, can notably differ from $2\pi f\mu$. Moreover, as is known from theory, the value of gravitational mass compensation also depends upon the so-called "density coefficient" — relationship $\frac{\mu}{\sigma - \bar{\mu}} = k$, where μ is the density of topographic mass and σ — density of mass of extended substrate (upper mantle); $\bar{\mu}$ — mass density of the crust, which need not coincide with topographic density μ. Our preliminary investigations based on materials of deep seismic sounding results provide a value for k that differs from the standard value 4.45 (for continents) and 2.73 (oceans) [6].

Therefore, if one calculates compensation according to the Erie system by known formulas for a limited plane layer having radius r, for average value H_r (within limits of 1 to 3 km) then the coefficient with H_r will equal from /7
0.047 $\frac{mgl}{m}$ to 0.092 $\frac{mgl}{m}$ depending upon the accepted values of r and k, while at the same time coefficient in topography $2\pi f\mu$ with standard values μ yield numbers 0.104-0.117 $\frac{mgl}{m}$ [7], i.e., error reaches up to several tens of percentage points. Here $T_0 = 33$ km is accepted as the normal crust value.

Proponents of the isostatic reduction of the Faye-Putnam system consider one of the reduction advantages to be the fact that the value of the corresponding anomaly and the method of finding the reduction itself do not clearly depend upon how the system of constructing the compensation masses is brought about [9]. From data presented it follows that the value of the reduction, and consequently of the anomaly, can strongly depend upon different variants of the accepted values of parameters: the radius of regionality and the relationship of crust density and substrate density.

The Faye formula (1) was also obtained by us in 1952 proceeding from concepts on the relationship of anomalies "in free air" and the altitude of observation point h in the Bouguer anomalies as they depend upon mean altitude H. The obtained anomaly was called the "local anomaly" by us [3, p. 54]. In 1957, formula (1) was again derived without any connection with the theory of isostatic compensation, laying within limitations of a region Δg of functions of only one h and placing it within a Taylor series by degrees (h-H).

$$\Delta g(h) = \Delta g(H) + \frac{\partial \Delta g}{\partial h}(h - H) + \cdots$$

where H is a certain mean value of h and derivative $\frac{\partial}{\partial h}\Delta g \approx 2\pi f \mu$ [4]. In 1958, de Graaf Hunter suggested it for his "model of the Earth with a smoothed topography" [14, pp. 1, 2]. Therefore, one can hardly justify calling formula (1) "the anomaly of de Graaf Hunter" [9, p. 208], as some people do these days. It is more correct to call this anomaly the Faye anomaly, since it was drawn up 80 years ago by Putnam [19]. For distinguishing it from the anomaly "in free air", we also frequently call it the "Faye anomaly" and it can be called the "isostatic Faye anomaly."

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Translated for the National Aeronautics and Space Administration under Contract No. NASw-2485 by Techtran Corporation, P.O. Box 729, Glen Burnie, Maryland, 21061; translator, Samuel D. Blalock Jr.